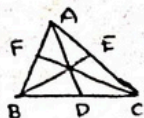


2-D Geometry

*) $\frac{PA}{PB} = \frac{QA}{QB} = \frac{m}{n}$

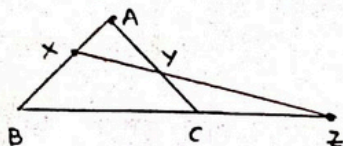
→ AP, AB, AQ are in HP

*) Ceva's theorem



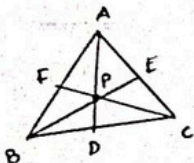
$$\frac{BD}{DC} \times \frac{EC}{EA} \times \frac{FA}{FB} = 1$$

*) Menelaus theorem



$$\frac{AX}{XB} \times \frac{BZ}{ZC} \times \frac{CY}{YA} = -1$$

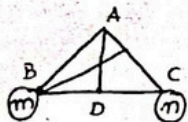
*) Van Aubel's theorem



$$\frac{AP}{PD} = \frac{AF}{FB} + \frac{AE}{EC}$$

*) Mass-point theorem

[Torque balancing]



$$\frac{BD}{DC} = \frac{m}{n}$$

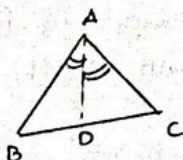
*) Sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$$

*) Cosine rule

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

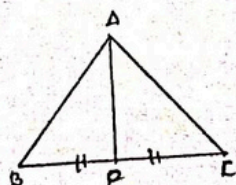
*) VABT



$$\frac{AB}{AC} = \frac{BD}{DC}$$

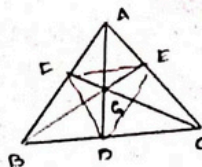
*) Length of median

$$2m_a^2 = b^2 + c^2 - a^2$$



Special points of Δ^c :

a) Centroid (G)



$$\frac{AG}{GD} = \frac{2}{1}$$

$$A_1(AQB) = A_1(BQC) = A_1(CQA)$$

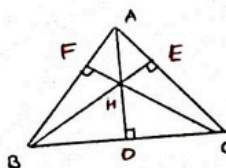
$$A_1(DEF) = \frac{1}{4} A_1(ABC)$$

$$AB^2 + BC^2 + CA^2 = 3[GA^2 + GB^2 + GC^2]$$

$$PA^2 + PB^2 + PC^2 = GA^2 + GB^2 + GC^2 + 3GP^2$$

* Centroid of ABC = Centroid of DEF P is any int pt.

b) Orthocentre (H)/(O):



$$\frac{AH}{HD} = \frac{\tan B + \tan C}{\tan A}$$

* In acute Δ^c , H lies inside.

* In obtuse Δ^c , H lies outside.

$$H = \left(\frac{\sum x_i \tan A}{\sum \tan A}, \frac{\sum y_i \tan A}{\sum \tan A} \right)$$

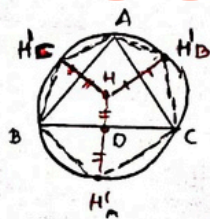
* orthocentre of ΔHBC is A

* orthocentre of ΔHAB is C

* circumradius of ΔABC and ΔHBC are equal.

*) orthocentre of ABC = incentre of ΔDEF

*) Image of orthocentre of Δ^c about any base lies on its circumcircle.

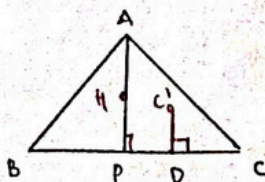


$$HD = DH'$$

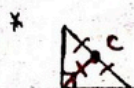
$$A_1 \text{ of Hexagon} = 2 A_1(ABC)$$

c) Circumcentre

$$O = \left(\frac{\sum x_i \sin 2A}{\sum \sin 2A}, \frac{\sum y_i \sin 2A}{\sum \sin 2A} \right)$$



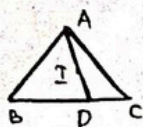
$$\frac{AO}{OD} = \frac{2}{1}$$



* Euler's line $HG:GS = 2:1$

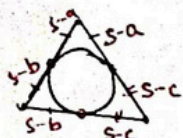
*) If circumcenter of a Δ^e is at $(0,0)$ then
Orthocenter is $H \equiv (x_1+x_2+x_3, y_1+y_2+y_3)$

Incentre



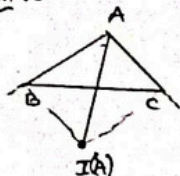
$$I \equiv \left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c} \right)$$

$$\frac{AI}{IO} = \frac{b+c}{a}$$



$$\Delta = rs$$

Ex-centre



$$I_A \equiv \left[\frac{-ax_1+bx_2+cx_3}{-a+b+c}, \frac{-ay_1+by_2+cy_3}{-a+b+c} \right]$$

Area of Δ^e and polygon

$$\Delta = \frac{1}{2} \begin{vmatrix} 1 & x_2 & y_2 \\ 1 & x_1 & y_1 \\ 1 & x_3 & y_3 \end{vmatrix} \quad (\text{mod})$$

$$\Delta = \frac{1}{2} \left| \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} + \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} + \begin{vmatrix} x_3 & y_3 \\ x_1 & y_1 \end{vmatrix} \right|$$

Translation of axis

*) Old + Old = New + New

*) Given second deg equation

$ax^2+2hxy+by^2+2gx+2fy+c=0$, to remove
 x, y terms (linear), $(0,0)$ must be shifted

$$\text{to } \left[\frac{hf-bg}{ab-h^2}, \frac{gh-af}{ab-h^2} \right]$$

Rotation

	X	Y
X	$\cos\theta$	$-\sin\theta$
Y	$\sin\theta$	$\cos\theta$

*) Gen second deg $\rightarrow xy$ term $\rightarrow$

$$\Rightarrow \tan 2\theta = \frac{2h}{a-b}$$

Locus

*) Pt equi from 2 fixed pts $\Rightarrow \perp^r$ bisector

*) Pt equi from 2 fixed lines $\Rightarrow$ angle bisector
(non-parallel)

Polar-coordinates

$$P(r_1, \theta_1)$$

$$Q(r_2, \theta_2)$$

$$PQ = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos(\theta_1-\theta_2)}$$

*) If all 3 vertices of Δ^e have rational
coordinates then such Δ^e can never be
equilateral.

*) The Δ^e formed by Excentres of ABC has
its ortho centre at Incentre of ABC.

$\Downarrow$

$$H \text{ of } (I_A I_B I_C) = I \text{ of } (ABC)$$

*)



3 $\rightarrow$ foot of $\perp^r$

3 $\rightarrow$ foot of medians

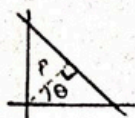
3 $\rightarrow$ midpoints of line
AH, BH, CH

$$*) \text{ONGC} \rightarrow ON:NG:GC = 3:1:2$$

$$OG:GC = 2:1$$

Straight Lines

*)



$$x \cos \theta + y \sin \theta = p$$

*)

$$\frac{x-x_1}{\cos \theta} = \frac{y-y_1}{\sin \theta} = r$$

*) Angle b/w 2 lines, $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

*) $\perp^r$ distance

$$\left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right| = d$$

*) foot of $\perp^r$

$$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-(ax_1 + by_1 + c)}{a^2 + b^2}$$

*) Image of a pt

$$\frac{h-x_1}{a} = \frac{k-y_1}{b} = \frac{-2(ax_1 + by_1 + c)}{a^2 + b^2}$$

*) distance b/w 2 parallel lines

$$\begin{aligned} ax + by + c_1 &= 0 \\ ax + by + c_2 &= 0 \end{aligned}$$

$$\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$$

*) $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0 \rightarrow$ doesn't just imply concurrent
*) parallel/concurrent

Family of Lines

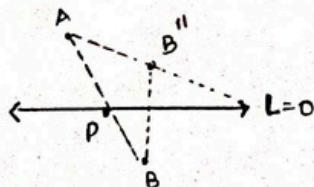
$$L_1 + \lambda L_2 = 0$$

Position of point

$\rightarrow$ make y coeff > 0
 $\rightarrow$ then $> 0 \rightarrow$ above
 $< 0 \rightarrow$ below.

Optimisation

Case-1)



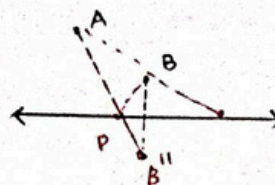
$(PA + PB)_{\min} \rightarrow$ POI of L & AB.

$(PA + PB)_{\max} \rightarrow$ Infinite

$(PA - PB)_{\min} \rightarrow '0'$

$(PA - PB)_{\max} \rightarrow AB''$

Case-2)



$(PA + PB)_{\min} \rightarrow AB''$

$(PA + PB)_{\max} \rightarrow$ Infinite

$(PA - PB)_{\min} \rightarrow '0'$

$(PA - PB)_{\max} \rightarrow AB$

Angle Bisectors

$$\frac{a_1 x + b_1 y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2 x + b_2 y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

* make $c > 0$ & then if $a_1 a_2 + b_1 b_2 > 0$
 $\rightarrow$ acute $\rightarrow '-'$
obtuse $\rightarrow '+'$

* make $c > 0$ & then bisector with '+' sign always contains the origin.

Pair of st lines

$$ax^2 + 2hxy + by^2 = 0 \quad (h^2 \geq ab)$$

$$\tan \theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

*) $a+b=0 \rightarrow \perp^r$

*) If $m_1 = m_2 \rightarrow$ coincidental

*) Eq of pair of lines $\perp^r$ to given lines

$$bx^2 - 2hxy + ay^2 = 0$$

*) pair of angle bisectors

$$\frac{x^2 - y^2}{a-b} = \frac{xy}{h}$$

*) Area of Δ

$$\begin{aligned} L_1 x + m_1 y + n_1 &= 0 \\ L_2 x + m_2 y + n_2 &= 0 \\ K_1 &= 0 \\ K_2 &= 0 \end{aligned}$$

$$\Delta = \left| \frac{(n_1 - n_2)(k_1 - k_2)}{L_1 m_2 - L_2 m_1} \right|$$