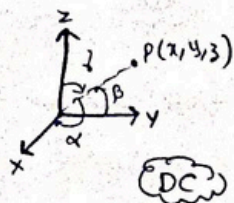


3D, PLANES

* The ratio in which the plane $P=0$ divides join of (x_1, y_1, z_1) and (x_2, y_2, z_2) is $\left(\frac{-P_1}{P_2}\right)$

(DC, DR) → just like slope in 2D



$$0 \leq \alpha, \beta, \gamma \leq \pi$$

$$\cos \alpha = \frac{x}{OP} = l$$

$$\cos \beta = \frac{y}{OP} = m$$

$$\cos \gamma = \frac{z}{OP} = n$$

$$\vec{r} = l\hat{i} + m\hat{j} + n\hat{k}$$

$$\Rightarrow l^2 + m^2 + n^2 = \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

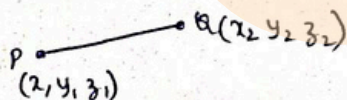
$$\Rightarrow \sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$$

DR

$$\frac{l}{a} = \frac{m}{b} = \frac{n}{c} = k \quad [\text{ratio's}]$$

$$\text{DC of } (a, b, c) \text{ is } \left(\frac{a}{\Sigma a^2}, \frac{b}{\Sigma b^2}, \frac{c}{\Sigma c^2}\right)$$

DR of line joining 2 points



$$\text{DR of line PQ} = (x_2 - x_1, y_2 - y_1, z_2 - z_1)$$

Angle between two lines

$$\cos \theta = \frac{|l_1 l_2 + m_1 m_2 + n_1 n_2|}{\sqrt{l_1^2 + m_1^2 + n_1^2} \sqrt{l_2^2 + m_2^2 + n_2^2}}$$

$l_i \rightarrow$ DC's

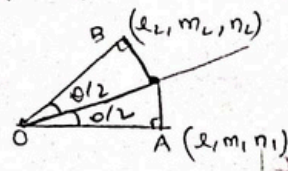
if DR's are given, then

$$\cos \theta = \frac{|a_1 a_2 + b_1 b_2 + c_1 c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$\perp \Rightarrow a \cdot b = 0 \Rightarrow a_1 a_2 + b_1 b_2 + c_1 c_2 = 0$$

$$\parallel \Rightarrow a \times b = 0, \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

* DC's of angular bisector



$$(l_1 + l_2, m_1 + m_2, n_1 + n_2)$$

$$\left(\frac{l_1 + l_2}{2 \cos \theta/2}, \frac{m_1 + m_2}{2 \cos \theta/2}, \frac{n_1 + n_2}{2 \cos \theta/2}\right) \text{ internal}$$

$$\left(\frac{l_1 - l_2}{2 \sin \theta/2}, \frac{m_1 - m_2}{2 \sin \theta/2}, \frac{n_1 - n_2}{2 \sin \theta/2}\right) \text{ external}$$

St Lines :-

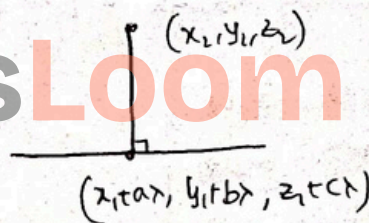
$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} \quad (\text{parametric form})$$

$$\vec{r} = (x_1 \hat{i} + y_1 \hat{j} + z_1 \hat{k}) + \lambda (a \hat{i} + b \hat{j} + c \hat{k}) \quad \text{vector form}$$

Angle between two lines

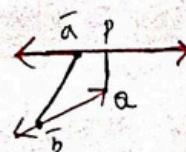
$$\cos \theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1| |\vec{n}_2|} \quad \left(\begin{array}{l} \delta = m_1 + \lambda n_1 \\ r = m_2 + \lambda n_2 \end{array} \right)$$

$\perp$ distance from a point to a line



$$\text{DC} \cdot \text{DC} = 0$$

Skew lines :-

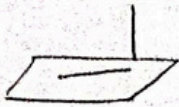


$$\text{vector along PQ} = \pm \frac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$$

Proj of AB on it

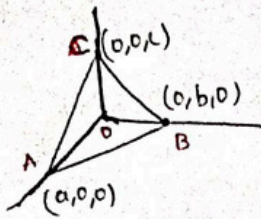
$$= \left| \frac{(b - a) \cdot (\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|} \right|$$

PLANES



$$\vec{r} \cdot \vec{n} = d$$

*



$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

(*) $Ad(ABC) = \sqrt{A_x^2 + A_y^2 + A_z^2}$

(*) foot of $\perp^r$ from origin on that plane is the orthocentre of $\triangle ABC$.

* Plane passing through 3 points

$$[AP \ AB \ AC] = 0$$

$$[\vec{r} \ \vec{b} \ \vec{c}] + [\vec{r} \ \vec{c} \ \vec{a}] + [\vec{r} \ \vec{a} \ \vec{b}] = [\vec{a} \ \vec{b} \ \vec{c}]$$

* Eq of plane passing through 3 given points

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

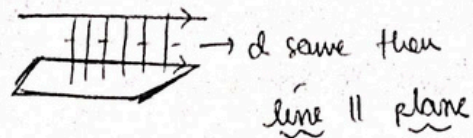
* Angle b/w 2 planes = $\cos \theta$

* Eq of plane through 1 point & parallel to 2 given lines

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} = 0$$

* line & plane angle = $\sin \theta$

*



* line is at const distance from 2 planes $\Rightarrow$ it's parallel to both planes

* mid point of 2 pts lie on a plane, it is not necessary that they are mirror images of each other.

* There are 7 planes which are equidistant from 4 non-collinear points.

* Eq of plane passing through point of $\otimes$ of planes $\Rightarrow P_1 + \lambda P_2$