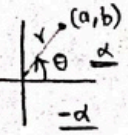
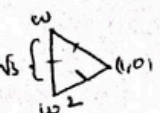


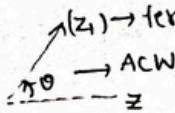
COMPLEX NUMBERS

- * $z_1 > z_2 \rightarrow$ meaning less (only real $\checkmark$)
- * $\arg(z) = \arg(z) = \theta \quad (-\pi, \pi)$
 $r = \sqrt{a^2 + b^2} = |z|$

- * $e^{ix} = \cos x + i \sin x$ * $z = x + iy = r(\cos \theta + i \sin \theta) = r e^{i\theta}$
- * $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$ (atleast one of them must $\in \mathbb{R}$)
- * $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2) + 2n\pi \quad (n \in \mathbb{Z})$
- * $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2) + 2n\pi \quad (n \in \mathbb{Z})$
- * $\log_e z = \log_e |z| + i \arg(z)$
- * $\operatorname{Re}(z) = \frac{z + \bar{z}}{2}$ * $\operatorname{Im}(z) = \frac{z - \bar{z}}{2i}$
- * $z = \bar{z} \Leftrightarrow$ pure real
- * $z = -\bar{z} \Leftrightarrow$ pure img
- * $|z|^2 = z \cdot \bar{z}$
- * $|z_1 z_2 - z_1| = |z_1| |z_2| - |z_1|$ * $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$
- Conj $\rightarrow$ mirror image abt Real axis.
- * $\overline{z_1 \pm z_2} = \bar{z}_1 \pm \bar{z}_2$ * $\overline{z^n} = (\bar{z})^n$
- * $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{(\bar{z}_1)}{(\bar{z}_2)}$ * $\overline{z_1 z_2} = \bar{z}_1 \cdot \bar{z}_2$
- * $\overline{z_1 z_2} = r_1 r_2 e^{-i(\theta_1 + \theta_2)}$ * $\overline{(\bar{z})} = z$
- Mod * $z_1^2 + z_2^2 = 0$ doesn't mean $z_1 = z_2 = 0$
- * $|z_1 - z_2| \rightarrow$ distance b/w 2 pts z_1 & z_2
- * $|z| = |\bar{z}| = |-z| = |- \bar{z}|$
- * $|az_1 + bz_2|^2 = (az_1 + bz_2)(a\bar{z}_1 + b\bar{z}_2)$
 $= a^2 |z_1|^2 + b^2 |z_2|^2 + 2ab \operatorname{Re}(\bar{z}_1 z_2)$
- * $-|z| \leq \operatorname{Re}(z), \operatorname{Im}(z) \leq |z|$
- * $z_1^3 + z_2^3 + z_3^3 = z_1 z_2 + z_2 z_3 + z_3 z_1 \Rightarrow z_1, z_2, z_3$
are vertices of an equilateral Δ^{lc} .

- Cube root of unity $z^3 = 1 \Rightarrow z = 1, \omega, \omega^2$

- * $\omega = \frac{-1 + \sqrt{3}i}{2}$ * $\omega^3 = 1$
- * $1 + \omega + \omega^2 = 0$
- * $\omega^2 = \frac{-1 - \sqrt{3}i}{2}$ * $\bar{\omega} = \omega^2, \bar{\omega^2} = \omega$
- * $\omega = \frac{1}{\omega^2}, \omega^2 = \frac{1}{\omega}$

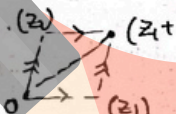
* Any CN whose $\frac{\operatorname{Re}(z)}{\operatorname{Im}(z)}$ or $\frac{\operatorname{Im}(z)}{\operatorname{Re}(z)}$ ratio is $\pm \sqrt{3}$ or $\pm \frac{1}{\sqrt{3}}$
then that CN (z) can be expressed in terms of ω

Argument:

- * $\arg$ of $(z_1 - z_2) \Rightarrow$ 
- * $\arg\left(\frac{z - z_1}{z - z_2}\right) = \alpha$


Solving Egⁿ

- S-1) First check $(0, \pm 1, \pm i)$
- S-2) Take Mod on B.S
- S-3) If S-2 doesn't work put $z = x + iy$ & simplify

- * 

$$* |z_1| \sim |z_2| \leq |z_1 + z_2| \leq |z_1| + |z_2|$$

DMT:

- * $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta \quad (\forall n \in \mathbb{N})$
- * If $n = \frac{p}{q}$ then $\cos n\theta + i \sin n\theta$ will still be one of the values.

N^{th} root of unity

$$z = (r e^{i\theta})^{1/n} = r^{1/n} \cdot (\cos \theta + i \sin \theta)^{1/n}$$

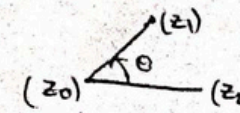
$$\bar{z} = r^{1/n} \left(\cos\left(\frac{2k\pi}{n}\right) + i \sin\left(\frac{2k\pi}{n}\right) \right) \quad (k \in 0, 1, \dots, n-1)$$

NOTE:-

- * $a^2 + b^2 + c^2 - ab - bc - ca = (a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)$
- * $a^3 + b^3 + c^3 - 3abc = (a + b + c)(a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)$
- * $a^3 + b^3 = (a + b)(a + b\omega)(a + b\omega^2)$
- * $a^3 - b^3 = (a - b)(a - b\omega)(a - b\omega^2)$

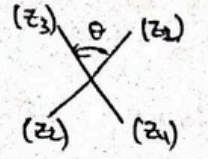
Rotation Theorem

* $\frac{z_1 - z_0}{|z_1 - z_0|} = \frac{z_2 - z_0}{|z_2 - z_0|} e^{i\theta}$

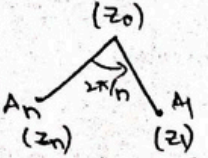


* $\frac{z_3 - z_1}{|z_3 - z_1|} = \frac{z_2 - z_1}{|z_2 - z_1|} e^{i\theta}$

(terminating vector) = (int v) × (rotation in cw by angle θ)



* $(z_1 - z_0) = (z_n - z_0) e^{i2\pi/n}$



Geometry

* Ar of Δ^e formed by $(z_1), (z_2), (z_3)$

$$= \frac{i}{4} \begin{vmatrix} z_1 & \bar{z}_1 & 1 \\ z_2 & \bar{z}_2 & 1 \\ z_3 & \bar{z}_3 & 1 \end{vmatrix}$$

N^{th} root : $z^n = 1$

* Sum of all roots = $1 + \alpha + \alpha^2 + \dots + \alpha^{n-1} = 0$

* Product of roots = $1 \cdot \alpha \cdot \dots \cdot \alpha^{n-1} = e^{i(n-1)\pi} = (-1)^{n-1}$

$\Rightarrow$ From $1 + \alpha + \alpha^2 + \dots + \alpha^{n-1} = 0$

IMP $1 + \sum_{k=1}^{n-1} \cos \frac{2k\pi}{n} = 0$, $\sum_{k=1}^{n-1} \sin \frac{2k\pi}{n} = 0$

$\sum_{k=0}^{n-1} \cos \frac{2k\pi}{n} = 0$, $\sum_{k=0}^{n-1} \sin \frac{2k\pi}{n} = 0$

* $z^n - 1 = (z-1)(z-\alpha)(z-\alpha^2) \dots (z-\alpha^{n-1})$

$\Rightarrow (z-\alpha)(z-\alpha^2) \dots (z-\alpha^{n-1}) = \frac{z^n - 1}{z-1} \rightarrow \lim_{z \rightarrow 1} = n$

$\Rightarrow \sin \frac{\pi}{n} \cdot \sin \frac{2\pi}{n} \cdot \dots \cdot \sin \frac{(n-1)\pi}{n} = \frac{n}{2^{n-1}}$

$\Rightarrow \cos \frac{\pi}{n} \cdot \cos \frac{2\pi}{n} \cdot \dots \cdot \cos \frac{(n-1)\pi}{n} = \begin{cases} 0 & \text{even} \\ \frac{1}{2^{n-1}} & \text{odd} \end{cases}$

* $z^n = 1 \{z_1, z_2, \dots, z_n\}$

$S_k = z_1^k + z_2^k + \dots + z_n^k = \begin{cases} 1 + \omega + \omega^2 + \dots + \omega^{n-1} & \text{if } k \neq n \\ 0 & \text{if } k = n \end{cases}$

$S_k = \begin{cases} 0 & k \neq n \\ n & k = n \end{cases}$

$n \nmid 3k \quad n = 3k$

Complex Slope & Real Slope

* Eq of st line : $a\bar{z} + \bar{a}z + b = 0$

joining $A(z_1)$ & $B(z_2)$

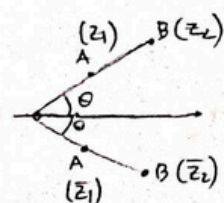
$a = i(z_1 - z_2)$

$b = 2i \operatorname{Im}(z_1 \bar{z}_2)$

$\Rightarrow$ Real slope $m = \frac{-\operatorname{Re}(a)}{\operatorname{Im}(a)}$

$\Rightarrow$ Complex slope = $\frac{z_2 - z_1}{\bar{z}_2 - \bar{z}_1}$

from eq, $m_c = \left(\frac{-a}{\bar{a}}\right)$



* If two lines with complex slopes ω_1, ω_2 are

$\rightarrow$ Parallel $\Leftrightarrow \omega_1 = \omega_2$

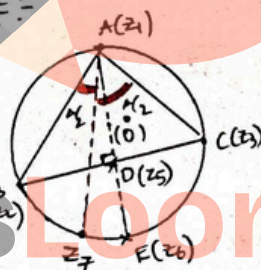
$\rightarrow \perp \Leftrightarrow \omega_1 + \omega_2 = 0$

* $\perp$ distance of (z_0) from $a\bar{z} + \bar{a}z + b = 0$

$= \frac{|a\bar{z}_0 + \bar{a}z_0 + b|}{2|a|}$

* Algebraic sum of distances = $\frac{nb}{2|a|}$ ($z^n = 1$) $\rightarrow n$ roots

NOTE :-



$z_5 = \frac{1}{2} \left[z_1 + z_2 + z_3 - \frac{z_2 z_3}{z_1} \right]$

$z_6 = \frac{-z_2 z_3}{z_1}$

$z_H = z_1 + z_2 + z_3$

$z_7^2 = z_2 z_3 \Rightarrow z_7 = \pm \sqrt{z_2 z_3}$ (cx & int)