



$$\frac{PS}{PM} = e$$

- $e = 1 \rightarrow$  parabola
- $0 < e < 1 \rightarrow$  ellipse
- $e \rightarrow 0 \rightarrow$  circle
- $e > 1 \rightarrow$  Hyperbola
- $e = \sqrt{2} \rightarrow$  rec hyp
- $e \rightarrow \infty \rightarrow$  pair of lines

$$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$$

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} \rightarrow \Delta = 0, h^2 \geq ab \rightarrow \text{Pair of lines}$$

$$\frac{h^2 - ab}{(a+b)^2} = \frac{e^2 - 1}{(2 - e^2)^2}$$

$$\left( \frac{bg - hf}{h^2 - ab}, \frac{af - gh}{h^2 - ab} \right)$$

- $\Delta \neq 0$ 
  - $h = 0, a = b$  (circle)
  - $h^2 = ab$  (Parabola)
  - $h^2 < ab$  (ellipse)
  - $h^2 > ab$  (hyperbola) \*  $a+b=0 \rightarrow$  rec hyp

| Circle                                                                   | Parabola                                     | Ellipse                                                                                  | Hyperbola                                            |
|--------------------------------------------------------------------------|----------------------------------------------|------------------------------------------------------------------------------------------|------------------------------------------------------|
| $x^2 + y^2 = a^2$                                                        | $y^2 = 4ax$                                  | $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$                                                  | $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$              |
| $xx_1 + yy_1 = a^2$                                                      | $yy_1 = 2a(x+x_1)$                           | $\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$                                                | $\frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$            |
| $y = mx \pm a\sqrt{1+m^2}$                                               | $y = mx + \frac{a}{m}$                       | $y = mx \pm \sqrt{a^2m^2 + b^2}$                                                         | $y = mx \pm \sqrt{a^2m^2 - b^2}$                     |
| $\left( \frac{\pm am}{\sqrt{1+m^2}}, \frac{\mp a}{\sqrt{1+m^2}} \right)$ | $\left( \frac{a}{m^2}, \frac{2a}{m} \right)$ | $\left( \frac{\mp am}{\sqrt{a^2m^2 + b^2}}, \frac{\pm b^2}{\sqrt{a^2m^2 + b^2}} \right)$ | $\left( \frac{\pm am}{c}, \frac{\pm b^2}{c} \right)$ |
| Dir $x^2 + y^2 = 2a^2$                                                   | $x = -a$ (directrix)                         | $x^2 + y^2 = a^2 + b^2$                                                                  | $x^2 + y^2 = a^2 - b^2$                              |
| Aux $x^2 + y^2 = a^2$                                                    | $x = 0$ (tgt at vertex)                      | $x^2 + y^2 = a^2$                                                                        | $x^2 + y^2 = a^2$                                    |
| $\frac{x-x_1}{x_1+g} = \frac{y-y_1}{y_1+f}$                              | $y = mx - 2am - am^3$                        | $\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2 - b^2$                                        | $\frac{a^2x}{x} + \frac{b^2y}{y} = a^2 + b^2$        |
| $2a$ (diameter)                                                          | $4a$                                         | $2b^2/a$                                                                                 | $2b^2/a$                                             |