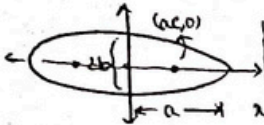


ELLIPSE

* $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$

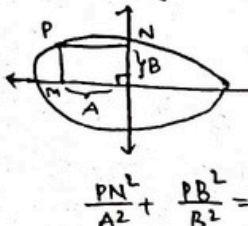
$\Rightarrow \Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} \neq 0, \quad h^2 < ab$

$\rightarrow \left(\frac{bg-hf}{h^2-ab}, \frac{af-gb}{h^2-ab} \right)$

*  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (a > b)$

* $c = \sqrt{a^2 - b^2}$; $ae = \sqrt{a^2 - b^2}$; $L = \frac{2b^2}{a}$

* $bx - ay + c = 0$ * $SP + S'P = 2a$

 $\frac{PN^2}{A^2} + \frac{PB^2}{B^2} = 1$ $x^2 + y^2 = a^2$ auxiliary circle

NOTE:-

* focal dis $\begin{cases} a+ex \\ a-ex \end{cases}$

* Eq of chord: $\frac{x}{a} \cos\left(\frac{\theta_1 + \theta_2}{2}\right) + \frac{y}{b} \sin\left(\frac{\theta_1 + \theta_2}{2}\right) = \cos\left(\frac{\theta_1 - \theta_2}{2}\right)$

* Tgt $c^2 = a^2 m^2 + b^2$

* $y = mx \pm \sqrt{a^2 m^2 + b^2}$

* $\left(\pm \frac{am}{c}, \pm \frac{b}{c} \right)$

* $S_1 = 0 \Leftrightarrow \frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$

* $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$

* $\left(\frac{a \cos\left(\frac{\theta_1 + \theta_2}{2}\right)}{\cos\left(\frac{\theta_1 - \theta_2}{2}\right)}, \frac{b \sin\left(\frac{\theta_1 + \theta_2}{2}\right)}{\cos\left(\frac{\theta_1 - \theta_2}{2}\right)} \right)$

* No. of tgts $D = h^2/a^2 + k^2/b^2 - 1 \rightarrow > 0$ 2 tgts

$\rightarrow = 0$ 1 tgt

* Angle b/w tgts $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| \rightarrow < 0$ No tgt

* combined eq of pair of tgts; $SS_{11} = S_1^2$

* Lr tgts (locus) $x^2 + y^2 = a^2 + b^2$ (Director circle)

* chord of contact; $\frac{2h}{a^2} + \frac{yk}{b^2} = 1$

* Eq of chord bisected; $S_1 = S_{11}$ at a given pt

* Normal

* $\frac{ax}{x_1} - \frac{by}{y_1} = a^2 - b^2$

* $a \operatorname{arccot} - b \operatorname{arccot} = a^2 - b^2$

* No. of Normals $a \operatorname{arccot} - b \operatorname{arccot} = a^2 - b^2$

put $\tan \theta_2 = T \rightarrow$ Biquadratic (max 4)

$\rightarrow$ maximum 4 Normals can be drawn

(feet of L's of Normals $\rightarrow$ Co-Normal points)

NOTE:-

$\rightarrow \tan \theta_1 \cdot \tan \theta_2 \cdot \tan \theta_3 \cdot \tan \theta_4 = -1$

$\rightarrow \theta_1 + \theta_2 + \theta_3 + \theta_4 = (2n+1)\pi$

$\rightarrow \sum \cos(\theta_1 + \theta_2) = 0$

$\rightarrow \sum \sin(\theta_1 + \theta_2) = 0$

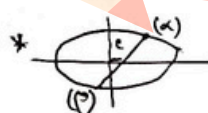
* $(x_1, x_2, x_3, x_4) \left(\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4} \right) = 4$

Properties :- & RESULTS:-

* Eccentric angle of extremities of LR $\theta = \tan^{-1}(b/ae)$

* $\frac{Ar(\Delta \text{ in ellipse})}{Ar(\Delta \text{ in aux circle})} = b/a$

eccentric angle



chord joining two pt $P(\alpha), Q(\beta)$ sub major at a dis of units from centre $\Rightarrow \tan \frac{\alpha}{2} \cdot \tan \frac{\beta}{2} = \frac{c-a}{c+a}$

* If the 2 chords are equidistant $\Rightarrow \tan \frac{\alpha}{2} \cdot \tan \frac{\beta}{2} \cdot \tan \frac{\gamma}{2} \cdot \tan \frac{\delta}{2} = 1$

* Sum of eccentric angles of con cyclic points $\Rightarrow 2n\pi$

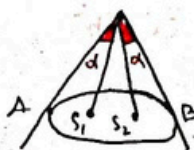
* Max area of Δ^c inscribed = $\frac{3\sqrt{3}ab}{4}$

* Min length made by any tgt = $|a+b|$

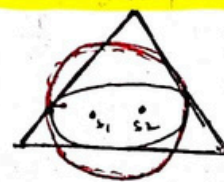
* Max distance of normal from c = $|a-b|$

Prop

*)



*)



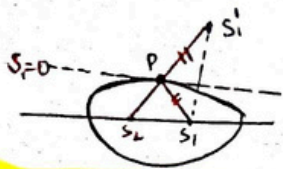
$S_1 \rightarrow$ orthocentre

$\Leftrightarrow$

$S_2 \rightarrow$ circumcentre

*) auxiliary circle = Nine point circle

(*)



$$PS_1 = a - ex_1$$

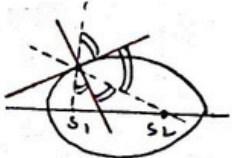
$$PS_2 = a + ex_2$$

⇒ Image of focus abt tgt at P is always collinear with POC of tgt and another focus

⇒ $S_2S'_1 = 2a$ ⇒ Locus of S'_1 is a circle with $r = 2a$

$$(x+ae)^2 + y^2 = 4a^2$$

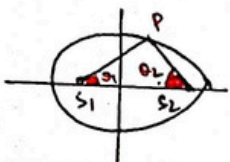
(*)



Tgt → External angle bisector

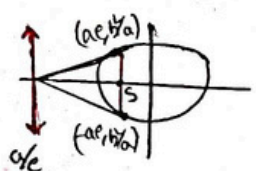
Nor → Internal angle bisector

(*)



$$\tan \frac{\theta_1}{2} \cdot \tan \frac{\theta_2}{2} = \frac{1-e}{1+e}$$

(*)



tgt of extremities of LR

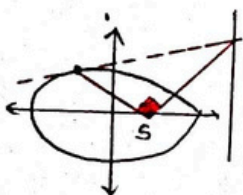
(*) the corresponding directrix

Position of tgt b/w curve

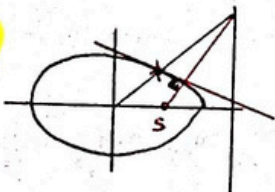
& directrix subtends

90° at corresponding focus

(*)



(*)



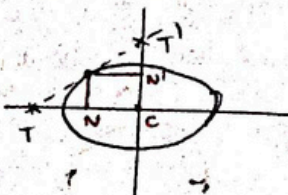
∠r from focus on any tgt & the line joining centre to POC (N) meet on corresponding directrix

(*)



$P_1P_2 = b^2$ Product of length of ∠r from foci upon tgt

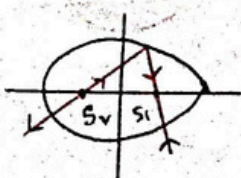
(*)



$$CN \cdot CT = a^2$$

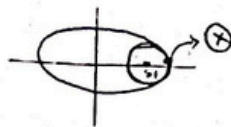
$$CN' \cdot CT' = b^2$$

(*)



(*)

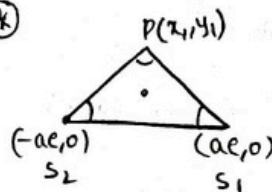
circle with S_1 as centre will touch ellipse only at vertex.



If $r > a(1-e)$ then

(2) pts

(*)



$$I \equiv (ex_1, \frac{ey_1}{1+e})$$

(*)



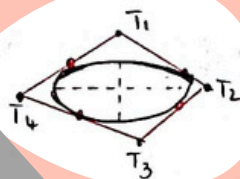
$$\frac{\cos \beta}{\cos \alpha} = e$$

(*)

If any chord PQ of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ subtends 90° at the centre, then

$$\frac{1}{CP^2} + \frac{1}{CQ^2} = \frac{1}{a^2} + \frac{1}{b^2}$$

(*)



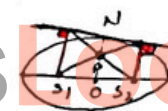
$$Ar(T_1T_2T_3T_4) = 4ab$$

(*)



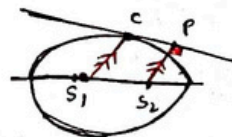
$$\Rightarrow e = \frac{1}{\sqrt{2}}$$

(*)



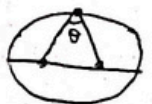
$$\Rightarrow NP = PO$$

(*)



$$SC \parallel S_2P$$

(*)



$$e = \sin^{-1} \theta/2$$