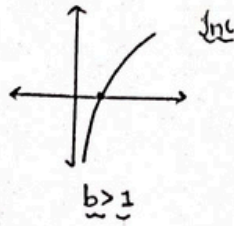
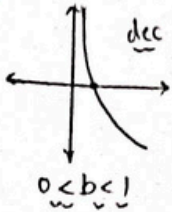


FUNCTIONS

Log(x):



$$a^{\log_b c} = b^{\log_a c}$$

$$a^{\log_a x} = x$$

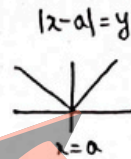
$$(a^{\log_b a})^n = (a^{\log_b a})^{1-n}$$

Mod

$$\# |x+y| = |x|+|y| \Leftrightarrow x, y \geq 0$$

$$\# |x-y| = |x|+|y| \Leftrightarrow x, y \leq 0$$

$$\# ||x|-|y|| \leq |x+y| \leq |x|+|y|$$

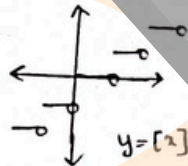


GIF, {x}

$$\# x-1 < [x] \leq x < [x]+1$$

$$\# [x+I] = I + [x] \quad I \in \text{Integer}$$

$$\# [x] + [-x] = \begin{cases} -1 & x \notin \mathbb{Z} \\ 0 & x \in \mathbb{Z} \end{cases}$$



$$\# [x+y] = \begin{cases} [x] + [y] \\ [x] + [y] + 1 \end{cases}$$

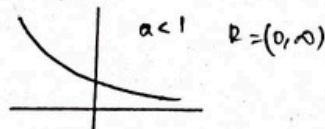
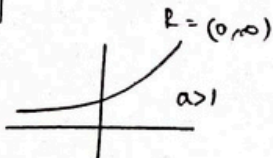
$$\# 0 \leq \{x\} < 1$$

$$\# \{x\} + \{-x\} = \begin{cases} 1 & x \notin \mathbb{Z} \\ 0 & x \in \mathbb{Z} \end{cases}$$

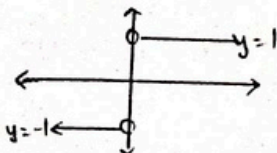
$$\# [x] + [x + \frac{1}{n}] + \dots + [x + \frac{n-1}{n}] = [nx] \quad n \in \mathbb{Z}$$

$$\# [\frac{x}{n}] + [\frac{x+1}{n}] + \dots + [\frac{x+n-1}{n}] = [x] \quad n \in \mathbb{Z}$$

a^x



Sgn(x)



$$\text{Sgn}(x) = \begin{cases} +1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

Function

(A) → (B)

def: every element in A must have an unique element mapped in B.

$$\# f(x) = \frac{x^2-1}{x-1} = \frac{(x+1)(x-1)}{(x-1)} = x+1 \quad [\text{wrong}]$$

↓
we cannot cancel out mother equation.

Even & odd

$$\# f(x) = f(-x) \quad \forall x \in D_f \quad \text{even} \rightarrow \text{sym abt } y\text{-axis}$$

$$\# f(x) = -f(-x) \quad \forall x \in D_f \quad \text{odd} \rightarrow \text{sym abt } (0,0)$$

* To say a function even/odd if $x=a$ is in D_f then $x=-a$ must be in domain.

Periodic functions

$$\# f(x+T) = f(x) \quad \forall x \in D_f \quad T > 0 \rightarrow \text{smallest } T \rightarrow \text{F.P.}$$

$$\# f(ax+b) \rightarrow T/|a|$$

$$\# P(x) = f(x) \pm g(x) \text{ (or)} \frac{f(x)-g(x)}{h(x)} \text{ or anything}$$

$$\text{Period of } P(x) = \text{LCM of } \{T_1, T_2, T_3\}$$

$$\# \text{LCM of } \{q_b, q_d, q_f\} = \frac{\text{LCM of } a, c, e}{\text{HCF of } b, d, f}$$

$$\# \text{LCM of } \mathbb{Q} \in \mathbb{Q}' \text{ isn't defined} \rightarrow \text{non-periodic.}$$

* Always check for funda period.

$$\# f(x) + f(x+T) = 0 \quad (T > 0) \quad \text{period} = 2T$$

$$\# f(x+a) + f(x+b) = 0 \rightarrow \text{period} = 2|a-b|$$

Types of functions:

* One-One (Injective)

→ every even deg poly is always many-one (Dr check)

→ If $f(x)$ is an even function, it must be many-one

→ use limits, monotonicity

* Onto (Surjective)

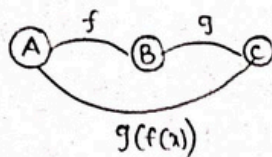
→ Range = Co-domain

$$\# \text{Range of } y = \frac{ax+b}{cx+d} = \mathbb{R} - \left\{ \frac{a}{c} \right\}$$

$$\# y = \frac{ax^2+bx+c}{ax^2-bx+c} \rightarrow \text{End of range is at } x = \pm \sqrt{\frac{c}{a}}$$

→ use graphs, diff, SOBS, AMGM ----

Composite functions



- * If $f(x)$ is one-one & $g(x)$ is one-one then $g \circ f \rightarrow 1-1$
- * If $f(x)$ is onto & $g(x)$ is onto then $g \circ f(x) \rightarrow$ onto
- * If $g \circ f(x)$ is one-one then $f(x)$ must be one-one
 $g(x)$ need not be one-one
- * If $g \circ f(x)$ is onto then $g(x)$ must be onto
 $f(x)$ need not be onto

* $f(x) = \frac{ax+b}{cx-a}$ then $f \circ f(x) = x$

* Use graph method to solve

Inverse functions: (both one-one & onto) $\rightarrow$ bijective

- * $f(x)$ & $f^{-1}(x)$ are always sym abt $y=x$
- * If $f(x)$ & $f^{-1}(x)$ are sym abt $y=-x$ then $f(x)$ must be odd
- * If (β, α) is a POI of $f(x)$ & $f^{-1}(x)$ then (β, α) must be another POI of $f(x)$ & $f^{-1}(x)$
- * All solutions of $f(x) = f^{-1}(x)$ must lie on $y=x$ (false)
- * $f(x) = f^{-1}(x)$ has a sol (α, β) and $f(x)$ is continuous then atleast one of $f(x) = f^{-1}(x)$ must lie on $y=x$.
- * $f(f^{-1}(x)) = x \quad \forall x \in B \quad [f: A \rightarrow B]$
 $f^{-1}(f(x)) = x \quad \forall x \in A \quad [f^{-1}: B \rightarrow A]$
 $\therefore f^{-1}(f(x)) = f(f^{-1}(x))$ [false]

Functional Equations:

$$\left. \begin{array}{l} f(x) + f(y) = f(xy) \\ f(x) - f(y) = f(xy) \end{array} \right\} f(x) = x \log_a x$$

2) $f(xy) = f(x) \cdot f(y) \rightarrow f(x) = x^n$

3) $f(x+y) = f(x) \cdot f(y) \rightarrow f(x) = a^x$

4) $f(x) + f(y) = f(x) \cdot f(y) \rightarrow \pm x^n + 1$

* For implicit functions, do partial diff $\left(\frac{\partial}{\partial x}\right)$