

HYPERBOLA

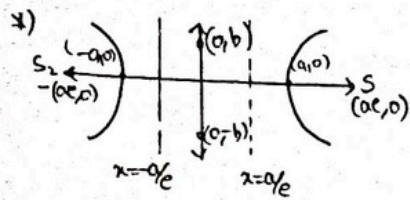
$ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$

$\Delta \neq 0, h^2 > ab$

$|S'P - SP| = 2a$

$e = \sqrt{1 + \frac{b^2}{a^2}}$

$S'P - SP = 2a$ single branch



$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

$x \rightarrow$ transverse axis

$y \rightarrow$ conjugate axis

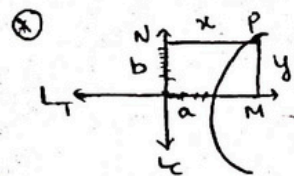
$LR = 2b^2/a$

focal distance: $\begin{cases} ex - a \\ ex + a \end{cases}$

Hyp is irrespective of a & b ($a > b$ or $b > a$)

$\Rightarrow$ It depends only on sign of x^2, y^2

Conjugate hyp $\frac{x^2}{a^2} - \frac{y^2}{b^2} = -1$



$\frac{PN^2}{a^2} - \frac{PM^2}{b^2} = 1$

$\frac{(LT)_L}{A^2} - \frac{(LT)_R}{b^2} = 1$

$\left(\frac{1}{e}\right)^2 + \left(\frac{1}{e_c}\right)^2 = 1$ $e_c \rightarrow$ Conj hyp

Auxiliary circle

$x^2 + y^2 = a^2$

$S_{11} > 0$ (inside)

$S_{11} < 0$ (outside)

NOTE:-

Eq of chord: $\frac{x}{a} \cos\left(\frac{\theta_1 - \theta_2}{2}\right) - \frac{y}{b} \sin\left(\frac{\theta_1 + \theta_2}{2}\right) = \cos\left(\frac{\theta_1 + \theta_2}{2}\right)$

$c^2 = a^2 m^2 - b^2$

$y = mx \pm \sqrt{a^2 m^2 - b^2}$

$\left(\pm \frac{a^2 m}{c}, \pm \frac{b^2}{c}\right)$

$S_1 = 0 \Rightarrow \frac{xx_1}{a^2} - \frac{yy_1}{b^2} = 1$

$\frac{x}{a} \sec \theta - \frac{y}{b} \tan \theta = 1$

No. of tgts from $y = mx \pm \sqrt{a^2 m^2 - b^2}$

quad in 'm'

$\rightarrow D > 0 \Rightarrow 2 \text{ tgts}$

$D = 0 \Rightarrow 1 \text{ tgt}$

$D < 0 \Rightarrow x \text{ tgt}$

$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

Director circle

$x^2 + y^2 = a^2 - b^2$

No. director circle $\left\{ \begin{array}{l} a^2 > b^2 \checkmark \\ a^2 = b^2 \\ a^2 < b^2 \times \end{array} \right.$

$\Rightarrow$ chord of contact $\frac{xh}{a^2} - \frac{yk}{b^2} = 1$

$\Rightarrow$ chord bisected $\frac{xh}{a^2} - \frac{yk}{b^2} = \frac{h^2}{a^2} - \frac{k^2}{b^2}$

Normal

$\frac{a^2 x_1}{x} + \frac{b^2 y_1}{y} = a^2 + b^2$

$ax \cos \theta + by \cos \theta = a^2 + b^2$

No. of Norm

$\downarrow$ put $t = \tan \theta/2$

Biquadratic...

$\rightarrow \theta_1 + \theta_2 + \theta_3 + \theta_4 = (2n+1)\pi$

$\Rightarrow \sum \cos(\theta_1 + \theta_2) = 0$

$\Rightarrow \sum \sin(\theta_1 + \theta_2) = 0$

$\ast (x_1 + x_2 + x_3 + x_4) \left[\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \frac{1}{x_4} \right] = 4$

$\ast (y_1 + y_2 + y_3 + y_4) \left[\frac{1}{y_1} + \frac{1}{y_2} + \frac{1}{y_3} + \frac{1}{y_4} \right] = 4$

Asymptotes:-

$y = mx + c$; $c = 0$
 $m = \pm b/a$ $b^2 = a^2 m^2$

$\ast \frac{x}{a} + \frac{y}{b} = 0, \frac{x}{a} - \frac{y}{b} = 0$

$\rightarrow$ Combined Eq, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$

Angle b/w asymptotes:

$\tan \theta = 2 \tan^{-1} b/a$

$\ast \tan \theta = 2 \sec^{-1}(e) = 2 \tan^{-1} b/a$

Rem:-

$\ast$ Eq of A - Eq of H = const

$\ast$ A of Conj H = A of H

$\ast$ H - A = A - CH

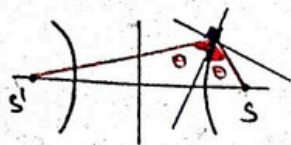
$\ast \ast$ Eq of asym $\rightarrow ax^2 + by^2 + \dots - c = 0$ } Hyp

take λ

make $|\det| = 0$

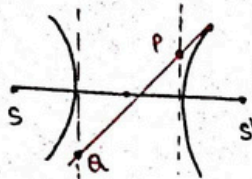
Properties:-

①



tgt & Normal
bisects the angle
b/w foci.

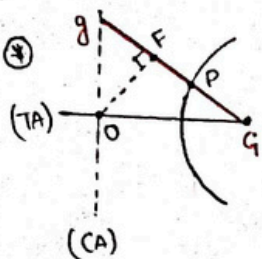
②



- P, S, S', A are concyclic
- $\angle PS'A = \angle PSA = 90^\circ$
- PA $\rightarrow$ diameter.

$\rightarrow$ POI of any tgt with
tgts at vertex are concyclic
with both the focus.

③



Normal at P meets TA at G,
CA at g.

F is the foot of $\perp$ from centre.

$$PF \cdot PG = b^2, \quad PF \cdot Pg = a^2$$

- ④ Product of $\perp$'s from any point on $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
to its asymptote = $\frac{a^2 b^2}{a^2 + b^2}$

- ⑤ If a chord intersect x-axis at (d, 0) then,
 $\tan \frac{\theta_1}{2} \cdot \tan \frac{\theta_2}{2} = \frac{a-d}{a+d}$

- ⑥ If it is focal chord through (ae, 0) then,

$$\tan \frac{\theta_1}{2} \cdot \tan \frac{\theta_2}{2} = \frac{1-e}{1+e}$$

$$(-ae, 0) \quad \tan \frac{\theta_1}{2} \cdot \tan \frac{\theta_2}{2} = \frac{1+e}{1-e}$$

- ⑦ If two chords of hyp are at equal distant
from origin, then.

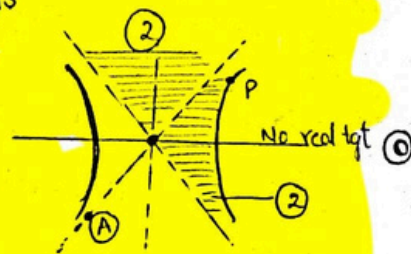
$$\tan \frac{\theta_1}{2} \cdot \tan \frac{\theta_2}{2} \cdot \tan \frac{\theta_3}{2} \cdot \tan \frac{\theta_4}{2} = 1$$

- ⑧ Quad formed from the foci of hyp & its conj
is always a square ($\sqrt{2}ae$)

- ⑨ Eq of common tgt hyp & its conj hyp

$$y = \pm x \pm \sqrt{a^2 - b^2}$$

No. of tgts



* Centre $\rightarrow$ No real tgt (2 Asym)

* P (on hyp) $\rightarrow 1$

* A (on Asym) $\rightarrow 1$

Rectangular Hyperbola

$$\Rightarrow x^2 - y^2 = a^2 \rightarrow xy = \frac{a^2}{2} \quad [\text{Rotating axis by } 45^\circ]$$

$$e = \sqrt{2}$$

* asymptotes are $\perp$ to each other

For $xy = c^2$,

→ Eq of directrix ; $x+y = \pm c\sqrt{2}$

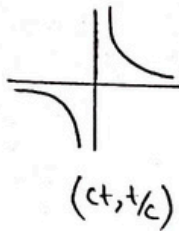
→ length of TA, CA ; $2\sqrt{2}c$

→ length of LA ; $2\sqrt{2}c$

→ Eq of conj rect ; $xy = -c^2$

→ vertices ; $(\pm c, \pm c)$

→ foci ; $(\pm\sqrt{2}c, \pm\sqrt{2}c)$



NOTES:-

→ slope = $-\frac{1}{t_1 t_2}$

* Eq of chord ; $x + y t_1 t_2 = c(t_1 + t_2)$

$$\text{tg} \theta + \frac{x}{x_1} + \frac{y}{y_1} = 2$$

* $x + y t^2 = 2ct$

* $\left(\frac{2ct_1 t_2}{t_1 + t_2}, \frac{2c}{t_1 + t_2} \right)$ {HM of x, y co-ordinates}

NOTES

* $xx_1 - yy_1 = x_1^2 - y_1^2$

* $y = t^2 x + \frac{c}{t} - ct^3$

* Eq of auxiliary circle $x^2 + y^2 = 2c^2$

Results:-

* If normal at t_1 meets curve again at t_2 then $t_1^3 \cdot t_2 = -1$.

* If three points A(t_1), B(t_2), C(t_3) are taken on $xy = c^2$, then orthocentre also lies on curve with co-ordinates $\left(\frac{-c}{t_1 t_2 t_3}, -ct_1 t_2 t_3 \right)$

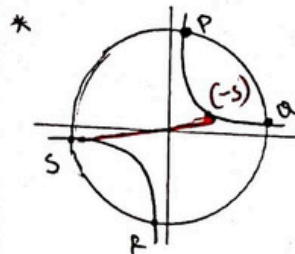
* Area of Δ formed by t_1, t_2, t_3 diametrically opp point

$$\left| \frac{c^2}{t_1 t_2 t_3} (t_1 - t_2)(t_2 - t_3)(t_3 - t_1) \right|$$

* If circle $x^2 + y^2 + 2gx + 2fy + c = 0$ meets $xy = c^2$ at 4 points, then ;

$$i) t_1 t_2 t_3 t_4 = 1$$

* ii) Mean of all the 4 x-co-ordinates of A of $\odot$ is same as the mean of x-coordinates of centred circle & Hyp.



orthocentre of $\Delta PQR \rightarrow (-s)$

* If 4 Normals are drawn to $xy = c^2$ meet at t_i

$$i) t_1 t_2 t_3 t_4 = -1$$

$$ii) (x_1 + x_2 + x_3 + x_4) = h$$

$$iv) (y_1 + y_2 + y_3 + y_4) = k$$

* If 4 focal chords of rec hyp are equal.

* If 3 normals drawn at t_1, t_2, t_3 meet on curve itself, then Centroid of ΔPQR coincides with centre of Hyp.

* If P(t_1), Q(t_2), R(t_3), S(t_4) are co-normal points,

→ orthocentre of PQR → S
QRS → P

* Any 2 $\perp$ chord for rec hyp $\rightarrow t_1 t_2 t_3 t_4 = -1$

* If any chord PQ of Hyp subtends 90° at R on same curve, then normal at R is parallel to chord joining PQ.

* If rec hyp passes through end points of diameter of circle, then that circle also passes through ends of diameter of Hyp.