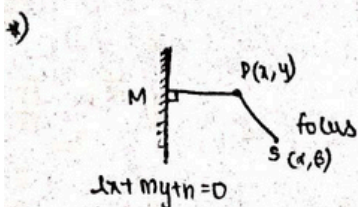
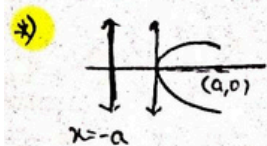


Parabola



$$(x-a)^2 + (y-0)^2 = c^2 \left| \frac{lx+my+n}{\sqrt{l^2+m^2}} \right|^2$$



$$y^2 = 4ax, \text{ latus rectum} = 4a$$

$$\text{focal distance} = |x_1 + a|$$

*) Slope of chord = $\frac{2}{t_1 + t_2}$ (at², 2at)

*) Focal chord ($t_1 t_2 = -1$)

→ one end is (at², 2at) & other is $(\frac{a}{t^2}, -\frac{2a}{t})$

→ $d_f = a(t_1 - t_2)^2$

$$= a(t + 1/t)^2$$

$$= 4a \csc^2 \theta \quad (\theta \rightarrow \text{Angle sub by chord at vertex})$$

$$= \frac{4a^3}{d^2} \quad (d \rightarrow \perp^r \text{ dis from vertex to fchord})$$

*) Tangent

$$y = mx + a/m \quad (a/m^2, 2a/m)$$

$$ty = x + at^2 \quad (at^2, 2at) \quad (s_1 = 0)$$

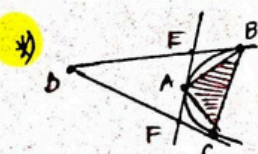
⇒ ⊗ of tgs → $[at_1 t_2, a(t_1 + t_2)]$

→ P → $\tan \theta = \frac{\sqrt{s_1}}{|x_1 + a|}$

⇒ chord of contact → $s_1 = 0$

*) Tgt at vertex → auxiliary circle (foot of $\perp^r$ s)

*) Directrix → director circle (90° b/w tgs)



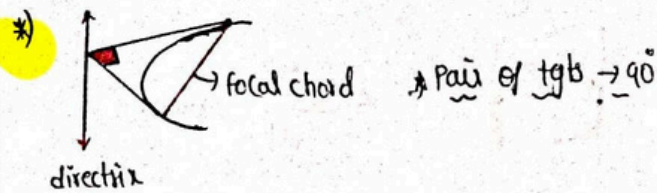
$$A(\triangle DEF) = \frac{1}{2} A(\triangle ABC)$$

*) orthocentre of DEF lies on directrix

*) Circumcircle of DEF always passes through focus of parabola (D, E, F, S are co-cyclic)

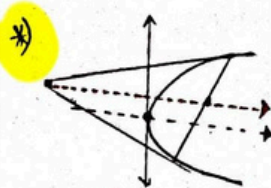
*) orthocentre of both Δ 's are equidistant from axis of parabola

*) line joining centroid of both Δ 's is || to axis of parabola

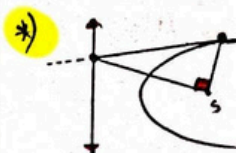


directrix

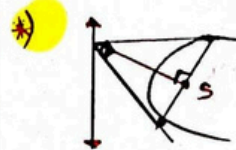
Pair of tgs → 90°



Pt of ⊗ of tgs & mid point of chord, line through these points is || to axis of P



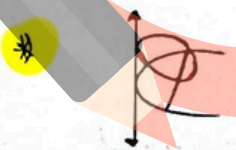
segment of tgt b/w POC & is ⊗ with directrix subtends 90° at focus.



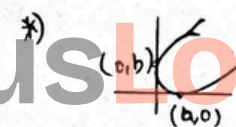
foot of $\perp^r$ of $\perp^r$ tgs to directrix on focal chord is the focus.



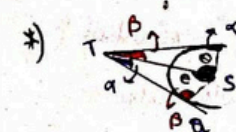
circle with focal chord as end of diameter always touches directrix.



circle with focal segment as diameter always touches tgt at vertex.



$$\sqrt{x/a} + \sqrt{y/b} = 1$$



$$ST^2 = SP \cdot SD$$

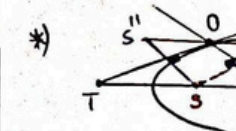
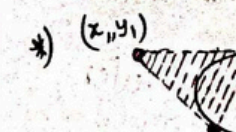


Image of focus abt any tgt (s'')
Image of focus with normal (s')
and point of contact (O)
are collinear.



$$\Delta = \frac{(s'')^3}{2a}$$

$$m_1 + m_2 = \frac{y_1}{x_1}$$

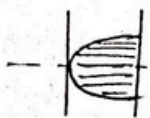
$$m_2 m_1 = \frac{a}{x_1}$$

*)



$$\Delta = \frac{d^3}{6(a)} \quad L = 4a$$

*)

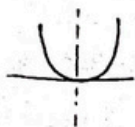


$$\Delta = \frac{8a^2}{3}$$

*) two lines $y = m_1x + c_1$ & $y = m_2x + c_2$ intersect co-ordinate axis at concyclic points, then:-

$$m_1 m_2 = 1$$

*)



$$x^2 = 4ay$$

$$\text{tgt eq} \rightarrow y = mx - am^2$$

*) Eq of Common tgt to $x^2 = 4ay$ & $y^2 = 4ax$

$$a^{1/3}x + b^{1/3}y + (a^{2/3}b^{1/3})^2 = 0$$



*) Normal:

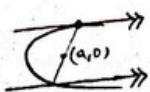
$$y = mx - 2am - am^3$$

*) Normal at one pt meet again at another point

$$t_1 = -\frac{2}{t} - t$$



*)



Tgt at one extremum of focal chord is parallel to Normal at other end

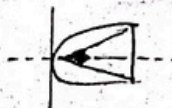
*) Length of subNormal = 2a

*) always constant

*) Normal at any pt is external angle bisector b/w focal dis & $\perp r$ on directrix

*) If normal chord subtends 90° at vertex

$$t = \pm\sqrt{2}$$



*) Length of shortest normal chord = $6a\sqrt{3}$

*) If PQ is normal chord, then min dis of O from vertex = $4\sqrt{6}a$

*) If normals at t_1 & t_2 meet parabola again,

$$t_1 t_2 = 2$$

*) Condition on Normals

$$4(h-2a)^3 > 27ak^2 \quad 3 \text{ distinct}$$

$$= 27ak^2 \quad 2 \text{ Equal, 1 dis}$$

$$< 27ak^2 \quad \text{only 1}$$

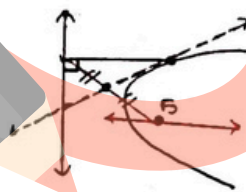
*) Position of point

$$y^2 - 4ax > 0 \quad \text{outside} (:::)$$

$$y^2 - 4ax < 0 \quad \text{inside} (:::)$$

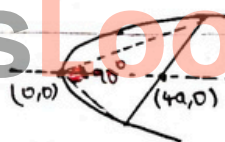


*)



Img of focus w.r.t tgt is the foot of $\perp r$ of POC on directrix

$$t_1 t_2 = -4$$



→ chord subtends 90° at vertex
→ always passes through $(4a, 0)$