

Quadratic Equations

$$* ax^2 + bx + c = a(x-\alpha)(x-\beta)$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\alpha + \beta = -b/a$$

$$\alpha\beta = c/a$$

$$|\alpha - \beta| = \sqrt{\Delta}/|a|$$

* For n^{th} deg poly;

$$a_0 x^n + a_1 x^{n-1} + \dots + a_n = 0$$

$$\Rightarrow \sum \alpha_i = -\frac{a_1}{a_0}$$

$$\Rightarrow \sum \alpha_1 \alpha_2 = a_2/a_0 \Rightarrow \sum \alpha_1 \alpha_2 \alpha_3 = -\frac{a_3}{a_0}$$

$$\Rightarrow \prod \alpha = (-1)^n \cdot \frac{a_n}{a_0}$$

NOTE:-

$$* a^3 + b^3 + c^3 - 3abc = (a^2 + b^2 + c^2 - ab - bc - ca)(a + b + c)$$

$$* \text{If } a + b + c = 0 \text{ then } a^3 + b^3 + c^3 = 3abc$$

$$* \text{If } a^3 + b^3 + c^3 = 3abc \text{ then,}$$

$$a + b + c = 0 \text{ (or) } a = b = c$$

$$* a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$* (a+b+c)^2 = abc(a+b+c) + (ab)^2 + (bc)^2 + (ca)^2$$

$$* f(x) = D(x) \cdot Q(x) + R(x)$$

$$\text{deg of } R(x) < \text{deg of } Q(x)$$

Nature of roots:

* If all co-eff are odd int, then roots cannot be rational.

* $ax^2 + bx + c$ $a, b, c \in \mathbb{Z} \Rightarrow$ roots must be Integers provided $\Delta = k^2$ $k \in \mathbb{Z}$

$$* ax^2 + bx^{n-1} + \dots + z = 0$$

$$\Rightarrow \text{Any possible rational root} = \pm \frac{(\text{All possible factors of } z)}{(\text{All possible factors of } a)}$$

Common root.

$$* (a_1 b_2 - a_2 b_1)(b_1 c_2 - b_2 c_1) = (c_1 a_2 - c_2 a_1)$$

$$* \alpha = \frac{a_1 c_2 - a_2 c_1}{a_2 b_1 - a_1 b_2}$$

Most imp

$$* ax^2 + bx + c \text{ (} a \neq 0 \text{)}$$

$$S_n = a^n + b^n \Rightarrow aS_n + bS_{n-1} + cS_{n-2} = 0$$

* If $P(x)$ & $Q(x)$ have common root

then $R(x) = \lambda P(x) + \mu Q(x) \rightarrow$ will also have same common root.

$$\lambda, \mu \in f(x)$$

* Both roots common [Given 1, but if they are irrational]

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

* Transformations,

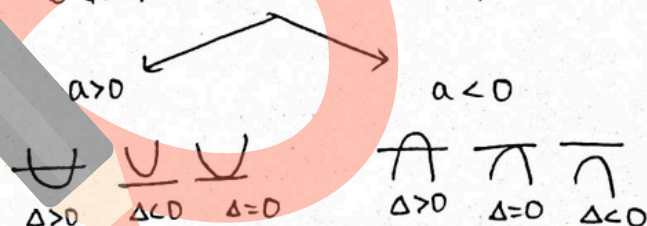
$$\alpha + k, \beta + k \rightarrow f(x-k) = 0$$

$$k\alpha, k\beta \rightarrow f(x/k) = 0$$

$$\alpha/k, \beta/k \rightarrow f(kx) = 0$$

$$1/\alpha, 1/\beta \rightarrow f(1/x) = 0$$

* Graphs:



$$* f(|x|) = \begin{cases} f(x) & x > 0 \\ -f(x) & x < 0 \end{cases}$$

$$* |f(x)| = \begin{cases} f(x) & f(x) > 0 \\ -f(x) & f(x) < 0 \end{cases}$$

* Signs

$$\rightarrow \text{check } -b/2a, \frac{4ac-b^2}{4a}$$

* Location of roots

$$\rightarrow f(\alpha) \cdot f(\beta) < 0 \Rightarrow \text{at least one root} \in (\alpha, \beta)$$

NOTE:-

① Less of roots

② Extraneous roots

③ Splitting Bi quad to quad

$$* (x^4 + x^2 + 1) = (x^2 + x + 1)(x^2 - x + 1)$$

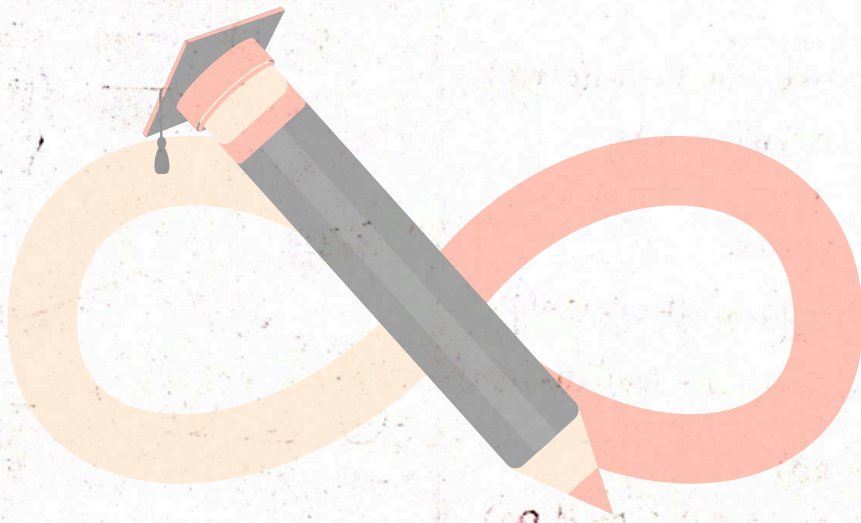
$$④ x + 1/x = t, \text{ sum, product, ...}$$

$$\begin{aligned}
 *) \quad & \left. \begin{aligned} ab(c+d)^2 + 2Pqd + Pqrs &= 0 \\ ab(c+e)^2 + 2Pqc + Pqrs &= 0 \end{aligned} \right\} \\
 & \text{then, } d, e \text{ are the roots of} \\
 & ab(x)^2 + 2Pqx + Pqs
 \end{aligned}$$

$$* \text{ Range of } \frac{\text{Quad}}{\text{Quad}} \rightarrow t = \dots$$

$$\hookrightarrow \frac{\text{Quad}}{\text{Cubic}} \rightarrow -\infty \text{ to } \infty$$

$$* \quad y = \frac{ax^2 + bx + c}{ax^2 - bx + c} \rightarrow \text{End of range is at } x = \pm \sqrt{c/a}$$



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