

## Sequence and Series

$$* S_n - S_{n-1} = t_n \quad (n \geq 2)$$

$$* S_1 = t_1$$

AP:

$$* \text{If } t_n = An + B \quad \text{Common diff} = A$$

$$* a_x = y \text{ \& } a_y = x \Rightarrow a_1 = x + y - x$$

$$* r^{\text{th}} \text{ term from end} \Rightarrow t_{n-r+1} = a + (n-r)d$$

$$* t_r + t_{n-r+1} = 2a + (n-1)d$$

$$* S_n = \frac{n}{2} (t_1 + t_{n+1-r}) \quad * a_n = S_{n+1} - S_n$$

$$* S_n = An^2 + Bn$$

$$* t_n = \frac{t_{n-1} + t_{n+1}}{2} = \frac{t_{n-m} + t_{n+m}}{2} = \dots$$

Selection of terms

$$\rightarrow (a-d), a, (a+d)$$

$$\rightarrow a-2d, a-d, a, a+d, a+2d$$

$$* AP \pm AP = AP$$

AM:

$$* AM = \frac{\sum a_n}{n}$$

$$* d = \frac{b-a}{n+1}$$

$$* \text{Sum of all AM's} = \frac{n}{2}(a+b)$$

$$* \text{Mean of } n \text{ AM's b/w } a \text{ \& } b = \frac{a+b}{2}$$

GP:

$$* S_n = \frac{a(r^n - 1)}{r - 1} \quad r \neq 1$$

$$* S_\infty = \frac{a}{1-r} \quad |r| < 1$$

$$* \text{If } a, b, c \text{ are in GP and also sides of } \Delta^{ic}, \text{ then } r \in \left( \frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}+1}{2} \right)$$

$$* t_1 t_n = t_2 t_{n-1} = \dots = \text{const}$$

$$* (t_n)^2 = (t_{n-k})(t_{n+k})$$

$$* a_1, a_2, \dots \text{ (GP) then } \log a_1, \log a_2, \dots \text{ (AP)}$$

GM

$$* r = \left( \frac{b}{a} \right)^{1/n+1}$$

$$* \text{GM of all } n \text{ GM's} = \sqrt[n]{ab}$$

$$* \text{Product of all } n \text{ GM's} = (\sqrt[n]{ab})^n$$

\* Any mean always lie b/w  $a$  \&  $b$

HP/HM

$$* HM = \frac{2ab}{a+b} = \frac{2}{\left(\frac{1}{a}\right) + \left(\frac{1}{b}\right)}$$

$$* d = \left( \frac{1}{b} - \frac{1}{a} \right) \frac{1}{n-1} \quad * S = \frac{n}{HM \text{ of } a \& b}$$

$$* a_1 a_2 + a_2 a_3 + \dots + a_{n-1} a_n = (n-1) a_1 a_n \quad \{a_1, a_2, \dots, a_n \text{ in HP}\}$$

AGP

$$ab, (a+d)br, (a+2d)br^2, \dots$$

$$* t_n = [a + (n-1)d] br^{n-1}$$

$$* S_\infty = \frac{ab}{1-r} + \frac{dbr}{(1-r)^2} \quad (|r| < 1)$$

$$* \text{To find } S_n, \Rightarrow r S_n \Rightarrow (1-r) S_n = \dots$$

Relation b/w means  $a, b \in \mathbb{R}^+$

$$* G^2 = AH$$

$$* x^2 - 2Ax + G^2 = 0 \quad (\text{roots are } a, b)$$

Sum

$$* \sum_{r=1}^n \sum_{m=1}^n A_r A_m = \left( \sum_{r=1}^n A_r \right) \left( \sum_{m=1}^n A_m \right) \quad \left\{ \begin{array}{l} r \& m \text{ are} \\ \text{ind on} \\ \text{each other} \end{array} \right\}$$

$$* \sum_{r=1}^n r = \frac{n(n+1)}{2}$$

$$* \sum_{r=1}^n r^2 = \frac{n(n+1)(2n+1)}{6}$$

$$* \sum_{r=1}^n r^3 = \left( \frac{n(n+1)}{2} \right)^2 = \left( \sum_{r=1}^n r \right)^2$$

Solving

$$\rightarrow \text{find } T_r \text{ [check properly \& verify]}$$

$$\rightarrow \text{then solve}$$

Method of diff

$$\rightarrow S_n = a + b + c + \dots + t_n$$

$$- S_n = -a - b - c - \dots - t_{n-1} - t_n$$

$$\rightarrow t_n = a + (AP) \Rightarrow \text{find } T_r$$

$$\text{then calculate } S_n$$

\* If first diff is in AP then,  $(k+1)^{\text{deg}}$

$$t_k = at^2 + bt + c \quad (a \neq 0) \quad t_n - t_1 = S_{n-1}$$

\* If first diff is in GP then,  $(k-1)^{\text{deg}}$

$$t_n = a + b \cdot R^n \quad (R \rightarrow \text{Common ratio of GP})$$

Telescopic:

→ manipulate  $f(r+1) - f(r)$

Inequalities:

\*  $AM \geq GM \geq HM$  [for  $R^+$ ]

\*  $AM = GM = HM \Leftrightarrow a = b$

Cauchy Schwartz

$$* (a_1^2 + a_2^2 + \dots + a_n^2)(b_1^2 + b_2^2 + \dots + b_n^2) \geq (a_1b_1 + a_2b_2 + \dots + a_nb_n)^2$$

Equality holds when  $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \dots = \frac{a_n}{b_n}$

NOTE:

$$* \frac{a_1^m + a_2^m + \dots + a_n^m}{n} \geq \left( \frac{a_1 + a_2 + \dots + a_n}{n} \right)^m \quad \begin{matrix} m < 0 \\ \text{or} \\ m > 1 \end{matrix}$$

$$* \frac{a_1^m + a_2^m + \dots + a_n^m}{n} \leq \left( \frac{a_1 + a_2 + \dots + a_n}{n} \right)^m \quad m \in (0, 1)$$

$$* RMS = \sqrt{\frac{a_1^2 + a_2^2 + \dots + a_n^2}{n}} \geq \frac{a_1 + a_2 + \dots + a_n}{n} \geq (a_1 a_2 a_3 \dots a_n)^{\frac{1}{n}}$$

\* If  $a, b, c$  are in AP, GP, HP then;

$$\frac{a-b}{b-c} = \frac{a}{k} \quad \begin{cases} k=a & \text{AP} \\ k=b & \text{GP} \\ k=c & \text{HP} \end{cases}$$

$$\textcircled{*} \lim_{n \rightarrow \infty} \frac{\sum r^a}{n^{a+1}} = \frac{1}{a+1}$$

\* Check whether given is GP or GM

$$-4 \times -1 \text{ all in GP} \Rightarrow +2$$

$$-4 \times -1 \rightarrow \text{GM} \Rightarrow -2$$

\* Positive quantities  $\rightarrow AM \geq GM \geq HM$