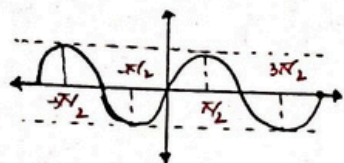


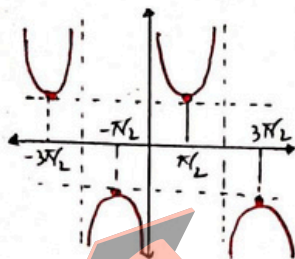
TRIGONOMETRY

$f(x)$	Domain	Range
$\sin x$	$\mathbb{R}$	$[-1, 1]$
$\cos x$	$\mathbb{R}$	$[-1, 1]$
$\tan x$	$\mathbb{R} - \{(2n+1)\pi/2\}$	$\mathbb{R}$
$\cot x$	$\mathbb{R} - \{(2n)\pi/2\}$	$\mathbb{R}$
$\sec x$	$\mathbb{R} - \{(2n+1)\pi/2\}$	$(-\infty, -1] \cup [1, \infty)$
$\csc x$	$\mathbb{R} - \{(n\pi)\}$	$(-\infty, -1] \cup [1, \infty)$

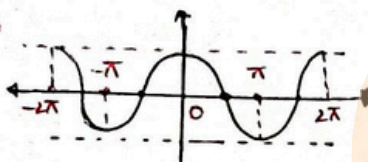
Graphs



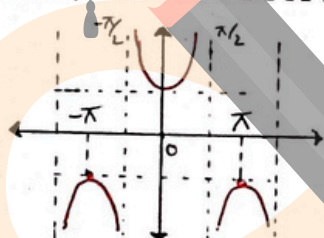
$$f(x) = \sin x$$



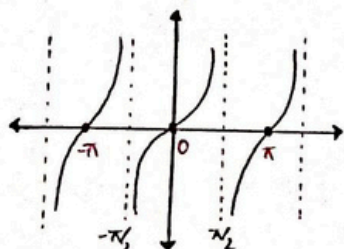
$$f(x) = \csc x$$



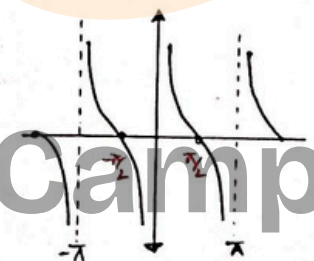
$$f(x) = \cos x$$



$$f(x) = \sec x$$



$$f(x) = \tan x$$



$$f(x) = \cot x$$

$$*) \tan x > x > \sin x \quad \forall x \in (0, \pi/2)$$

$$*) x > \sin x \quad \forall x \in \mathbb{R}^+$$

Comp angles :

$$*) \sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$*) \sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$*) \cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$*) \cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$*) \tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$*) \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$*) \sin(A+B) \cdot \sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 A$$

$$*) \cos(A+B) \cdot \cos(A-B) = \cos^2 A - \sin^2 B = \cos^2 B - \sin^2 A$$

$$*) \sin(A+B+C) = \cos A \cos B \cos C (\Sigma \tan A - \prod \tan A)$$

$$*) \cos(A+B+C) = \cos A \cos B \cos C (1 - \Sigma \tan A \tan B)$$

$$*) \tan 3\theta = \frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta}$$

$$*) \tan(n\theta) = \frac{n_1 \tan \theta - n_3 \tan^3 \theta + \dots}{1 - n_2 \tan^2 \theta + n_4 \tan^4 \theta - \dots}$$

Multiples & Sub-multiples :

$$*) \sin 2\theta = 2 \sin \theta \cos \theta = \frac{2 \tan \theta}{1 + \tan^2 \theta}$$

$$*) \cos 2\theta = 2 \cos^2 \theta - 1 = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

$$*) \tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$*) \sin 3\theta = 3 \sin \theta - 4 \sin^3 \theta$$

$$\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$$

$$*) \sin \theta \cdot \sin(60-\theta) \cdot \sin(60+\theta) = \frac{1}{4} \sin 3\theta$$

$$\cos \theta \cdot \cos(60-\theta) \cdot \cos(60+\theta) = \frac{1}{4} \cos 3\theta$$

$$\tan \theta \cdot \tan(60-\theta) \cdot \tan(60+\theta) = \tan 3\theta$$

$$\tan \theta + \tan(60-\theta) + \tan(60+\theta) = 3 \tan 3\theta$$

$$*) (1 + \tan A)(1 + \tan B) = 2 \Rightarrow A+B = 45^\circ$$

$$\Sigma \tan A = \prod \tan A \Rightarrow \Sigma A = n\pi$$

$$\Sigma \tan A \tan B = 1 \Rightarrow \Sigma A = (2n+1)\pi/2$$

$$*) \frac{1 + \cos \theta}{2} = \cos^2 \theta/2 \quad *) \frac{1 - \cos \theta}{2} = \sin^2 \theta/2$$

Transformations

$$*) \sin C + \sin D = 2 \sin \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right)$$

$$\sin C - \sin D = 2 \cos \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right)$$

$$\cos C + \cos D = 2 \cos \left(\frac{C+D}{2} \right) \cos \left(\frac{C-D}{2} \right)$$

$$\cos C - \cos D = -2 \sin \left(\frac{C+D}{2} \right) \sin \left(\frac{C-D}{2} \right)$$

$$*) \cos \theta \cdot \cos 2\theta \cdot \dots \cos 2^{n-1}\theta = \frac{\sin 2^n \theta}{2^{n-1} \sin \theta}$$

$$*) \sin \alpha + \sin(\alpha + \beta) + \dots \text{ (AP) } = \frac{\sin(\frac{\alpha + \beta}{2}) \cdot \sin(\frac{n\beta}{2})}{\sin(\frac{\beta}{2})}$$

$$*) \cos \alpha + \cos(\alpha + \beta) + \dots \text{ (AP) } = \frac{\cos(\frac{\alpha + \beta}{2}) \sin(\frac{n\beta}{2})}{\sin(\frac{\beta}{2})}$$

$$*) \cot \theta - \tan \theta = 2 \cot 2\theta$$

$$*) \cot \theta + \tan \theta = 2 \operatorname{cosec} 2\theta$$

$$*) \sum_{r=1}^n \frac{1}{\sin 2^{r-1}\theta} = \cot 2\theta - \cot 2^n \theta$$

$$*) \tan A + 2 \tan 2A + \dots + 2^{n-1} \tan 2^{n-1}A + 2 \cot 2^n A = \cot A$$

$$*) \text{ In any } \triangle ABC, \cos A + \cos B + \cos C \leq \frac{3}{2}$$

$$*) \sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$$

$$*) \sin 2A + \sin 2B - \sin 2C = 4 \cos A \cos B \sin C$$

$$*) \cos 2A + \cos 2B + \cos 2C = -1 - 4 \cos A \cos B \cos C$$

$$*) \cos A + \cos B + \cos C = 1 + 4 \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}$$

$$*) \cos \frac{\pi}{4} \cdot \cos \frac{2\pi}{4} \cdot \cos \frac{4\pi}{4} = -1/8$$

$$*) \cos \frac{2\pi}{4} + \cos \frac{4\pi}{4} + \cos \frac{6\pi}{4} = -1/2$$

$$*) \cos \frac{\pi}{4} + \cos \frac{3\pi}{4} + \cos \frac{5\pi}{4} = -1/2$$

$$*) \cos \frac{\pi}{4} \cdot \cos \frac{3\pi}{4} \cdot \cos \frac{5\pi}{4} = -1/8$$

$$*) \cos \frac{5\pi}{4} \cdot \cos \frac{2\pi}{4} \cdot \cos \frac{3\pi}{4} = -1/8$$

$$*) \tan^2 \frac{\pi}{4}, \tan^2 \frac{3\pi}{4}, \tan^2 \frac{5\pi}{4} \text{ are roots of } x^3 - 21x^2 + 35x - 7 = 0$$

standard :

$$*) \sin 18^\circ = \frac{\sqrt{5}-1}{4} = \cos 72^\circ$$

$$*) \cos 18^\circ = \frac{\sqrt{10+2\sqrt{5}}}{4} = \sin 72^\circ$$

$$*) \sin 54^\circ = \frac{\sqrt{5}+1}{4} = \cos 36^\circ$$

$$*) \cos 54^\circ = \frac{\sqrt{10-2\sqrt{5}}}{4} = \sin 36^\circ$$

* Range of $a \cos \theta + b \sin \theta + c$:

$$c - \sqrt{a^2 + b^2} \leq a \cos \theta + b \sin \theta + c \leq c + \sqrt{a^2 + b^2}$$

* Gen sol :

$$\sin \theta = \sin \alpha \Rightarrow \theta = n\pi + (-1)^n \alpha$$

$$\cos \theta = \cos \alpha \Rightarrow \theta = 2n\pi \pm \alpha$$

$$\tan \theta = \tan \alpha \Rightarrow \theta = n\pi + \alpha$$