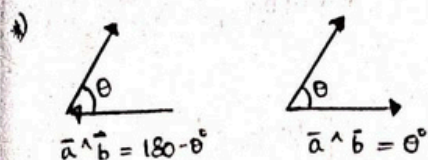


VECTORS



* Collinear $\Rightarrow \vec{A} = \lambda \vec{B}$

* Non-collinear but still $x\vec{a} + y\vec{b} = 0 \Rightarrow x=y=0$ (must)

* Co-planar $\Rightarrow \vec{c} = \lambda\vec{a} + \mu\vec{b}$

$\Rightarrow \vec{a} \cdot (\vec{b} \times \vec{c}) = 0$ [vol=0]

$\Rightarrow \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0$

* 4 points Co-planar

$[AB \ AC \ AD] = 0$

* $\vec{a} + \vec{b} = \vec{c}$

* $\vec{I} \equiv \frac{(BC)\vec{a} + (CA)\vec{b} + (AB)\vec{c}}{AB+BC+CA}$
 $\vec{H} = \frac{(\tan A)\vec{a} + (\tan B)\vec{b} + (\tan C)\vec{c}}{\tan A + \tan B + \tan C}$

$\Rightarrow \Delta = \frac{1}{2} |(\vec{b} \times \vec{c}) + (\vec{a} \times \vec{b}) + (\vec{c} \times \vec{a})|$

$\Rightarrow$ Unit Vector $\perp^x$ to plane of ΔABC $= \frac{(\vec{a} \times \vec{b}) + (\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a})}{2\Delta}$

* $\vec{R} = \vec{a} + \vec{b} \Rightarrow |\vec{R}| = \sqrt{a^2 + b^2 + 2ab \cos \theta}$

* Vector along angle bisector,

int $\rightarrow \frac{\vec{a} + \vec{b}}{2 \cos \frac{\theta}{2}}$ ext $\rightarrow \frac{\vec{a} - \vec{b}}{2 \sin \frac{\theta}{2}}$

* Dot Product:

$\vec{a} \cdot \vec{b} = ab \cos \theta \Rightarrow \theta = \cos^{-1} \left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \right)$

* $\vec{a} \cdot \vec{a} = a^2$

* $\vec{a} \cdot \vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}$

* $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

Projection:

Projection of $\vec{a}$ on $\vec{b}$
 $= \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \Rightarrow \text{mag} \times \text{Unit vector along } \vec{b}$
 $a \cos \theta$

CROSS PRODUCT:

$\vec{a} \times \vec{b} = (|\vec{a}| |\vec{b}| \sin \theta) \cdot \hat{n}$

* $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

* $\vec{a} \times \vec{b} = 0 \Rightarrow \theta = 0^\circ, 180^\circ$ (Parallel ...)

* $\Delta = \frac{1}{2} |\vec{a} \times \vec{b}|$

* $\Delta = |\vec{a} \times \vec{b}|$

* Gen, Area of Quad $= \frac{1}{2} |\vec{d}_1 \times \vec{d}_2|$

* $\vec{a} \cdot \vec{b} = 0$ & $\vec{a} \cdot \vec{c} = 0$ then $\vec{a} \parallel (\vec{b} \times \vec{c})$

* If $\vec{a} + \vec{b} + \vec{c} = 0$ (Cyclic)
 $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

* Linearly dependent Vectors:-

$\vec{r} = x_1 \vec{a}_1 + x_2 \vec{a}_2 + \dots + x_n \vec{a}_n$

If atleast one of $x_i \neq 0$ & $i=1, 2, \dots, n$ then they are called LDV

* 2 Collinear vectors are always LD.

* 2 non-collinear vectors are always LIND

* 3 co-planar vectors $\rightarrow$ LD

* 3 Non co-planar vectors $\rightarrow$ LIND

* More than 3 vectors $\rightarrow$ LD

Vector Triple product

$$\underline{\vec{a} \times (\vec{b} \times \vec{c})} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$$

$\perp^r$ to $\vec{a}$ & $\parallel^r$ to both $\vec{b}, \vec{c}$

Scalar Triple Product

$$[\vec{a} \ \vec{b} \ \vec{c}] = \vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c}$$

* Dot & Cross can be exchanged $\uparrow$

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{c} \cdot (\vec{a} \times \vec{b})$$

$$[\vec{a} \ \vec{b} \ \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

$$[\vec{a} \ \vec{b} \ \vec{c}] = [\vec{c} \ \vec{a} \ \vec{b}] = [\vec{b} \ \vec{c} \ \vec{a}]$$

* All prop of Det can be applied

* $[a \ b \ c] \rightarrow$ vol of parallelepiped

$$[\vec{a} \ \vec{b} \ \vec{c}] = |\vec{a}| |\vec{b}| |\vec{c}| \sin \theta \cos \alpha$$

$$[a \ b \ c]_{\max} = abc$$

$$[\vec{a} + \vec{b} \ \vec{b} + \vec{c} \ \vec{c} + \vec{a}] = 2[\vec{a} \ \vec{b} \ \vec{c}]$$

$$[\vec{a} - \vec{b} \ \vec{b} - \vec{c} \ \vec{c} - \vec{a}] = 0$$

$$[\vec{a} \times \vec{b} \ \vec{b} \times \vec{c} \ \vec{c} \times \vec{a}] = [\vec{a} \ \vec{b} \ \vec{c}]^2$$

$$[\vec{a} \times (\vec{b} \times \vec{c}) \ \vec{b} \times (\vec{c} \times \vec{a}) \ \vec{c} \times (\vec{a} \times \vec{b})] = 0$$

$$[a \ b \ c]^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{b} \cdot \vec{a} & \vec{b} \cdot \vec{b} & \vec{b} \cdot \vec{c} \\ \vec{c} \cdot \vec{a} & \vec{c} \cdot \vec{b} & \vec{c} \cdot \vec{c} \end{vmatrix}$$

$$[\vec{a} \ \vec{b} \ \vec{c}] \cdot [\vec{l} \ \vec{m} \ \vec{n}] = \begin{vmatrix} \vec{a} \cdot \vec{l} & \vec{a} \cdot \vec{m} & \vec{a} \cdot \vec{n} \\ \vec{b} \cdot \vec{l} & \vec{b} \cdot \vec{m} & \vec{b} \cdot \vec{n} \\ \vec{c} \cdot \vec{l} & \vec{c} \cdot \vec{m} & \vec{c} \cdot \vec{n} \end{vmatrix}$$

$$* [\vec{x}_1 \vec{a} + \vec{y}_1 \vec{b} + \vec{z}_1 \vec{c} \ \vec{x}_2 \vec{a} + \vec{y}_2 \vec{b} + \vec{z}_2 \vec{c} \ \vec{x}_3 \vec{a} + \vec{y}_3 \vec{b} + \vec{z}_3 \vec{c}]$$

$$= \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix} [a \ b \ c]$$

$$\text{Ex: } [-2\vec{a} + 3\vec{b} \ 2\vec{a} + 3\vec{b} + \vec{c} \ \vec{b} - \vec{c}]$$

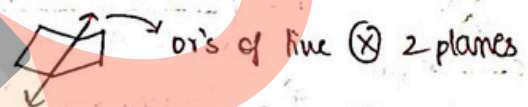
$$= \begin{vmatrix} -2 & 3 & 0 \\ 2 & 3 & 1 \\ 0 & 1 & -1 \end{vmatrix} [a \ b \ c]$$

$\xrightarrow{\text{Det}}$

* Lagrange's Identity :-

$$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = \begin{vmatrix} \vec{a} \cdot \vec{c} & \vec{a} \cdot \vec{d} \\ \vec{b} \cdot \vec{c} & \vec{b} \cdot \vec{d} \end{vmatrix}$$

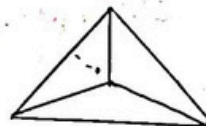
↓
Geometrical Meaning:



$$* [\vec{a} \ \vec{b} \ \vec{c}] > 0 \rightarrow \text{RHS}, < 0 \text{ LHS}$$

* If $\vec{a} \times \vec{b} = \vec{c}$, $\vec{b} \times \vec{c} = \vec{a}$, $\vec{c} \times \vec{a} = \vec{b}$
then $\vec{a}, \vec{b}, \vec{c}$ forms orthogonal triad.

Tetrahedron:



$$* \text{Vol} = \frac{1}{6} [\vec{a} \ \vec{b} \ \vec{c}] = \frac{1}{3} (\text{base} \times \text{height})$$

$$* \text{Centroid} \left(\frac{\sum x_i}{4}, \frac{\sum y_i}{4} \right)$$